The Undercut Phenomenon

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QUESTION

"It appears that undercut can be eliminated in some cases but in most cases, the elimination of undercut, for example by increasing the root fillet radii of a pinion, results in performance problems in the operation with its mate. My question is, when can I eliminate undercut and why is it not possible in most cases?"

Expert response provided by Dr. Hermann J. Stadtfeld

Introduction

The first questions are, where does the undercut come from and which physical or kinematical effects lead to undercut? Then the following questions are, can it be avoided and how, or can it be avoided only in certain cases?

A comparison between a tooth profile without undercut and a similar profile with undercut is shown in Figure 1. The profile at the left side of Figure 1 shows a healthy profile with a root fillet radius which blends perfectly tangential with the involute profile. In contrast, the right-side graphic in Figure 1 shows a severe undercut resulting in a ridge on both sides of the tooth which weakens the root and reduces the amount of profile depth, where the mating tooth can mesh.

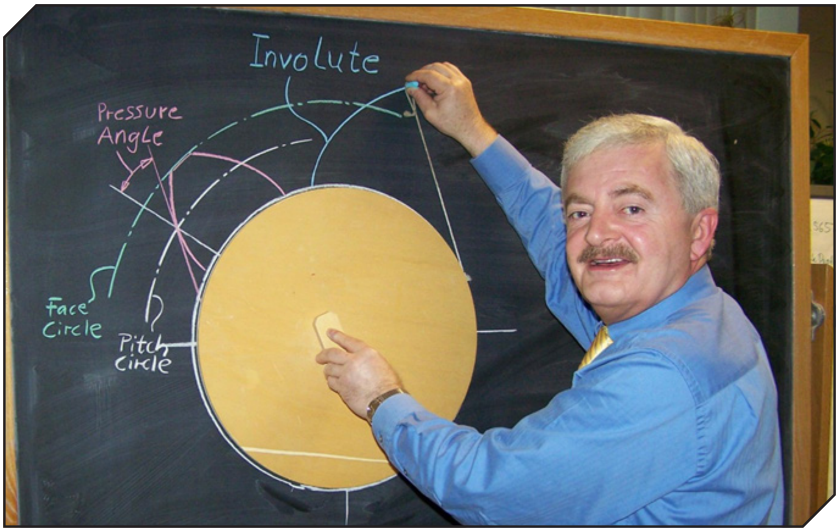

Figure 2 Involute development.

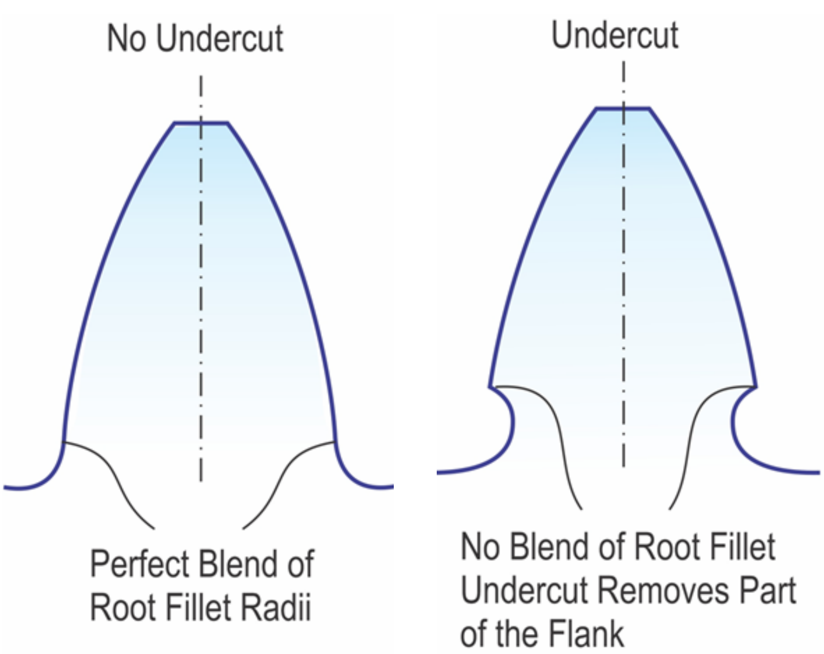

Figure 1 No undercut and undercut comparison.

Rule of Involute Generation

The term "undercut" comes from "undercutting," which is a hollowing out of an area because the cutting take place below a certain depth, where no Involute exists. In Figure 2 the author of this article demonstrates a classroom development of an involute by unrolling a cord from a disk. The disk holds here the place of the base circle and several geometric laws can be demonstrated with an involute development. It appears logical that the chalk which is used to draw the involute cannot draw anything below the base circle. In other words, no part of the involute exists below the base circle (or inside of the disk). This is because of the definition of the involute function and the resulting restrictions in the mechanics of drawing an involute.

The drawing in Figure 2 also shows that the unrolling of the cord can be continued into infinity. If a particular gear design has a pressure angle of

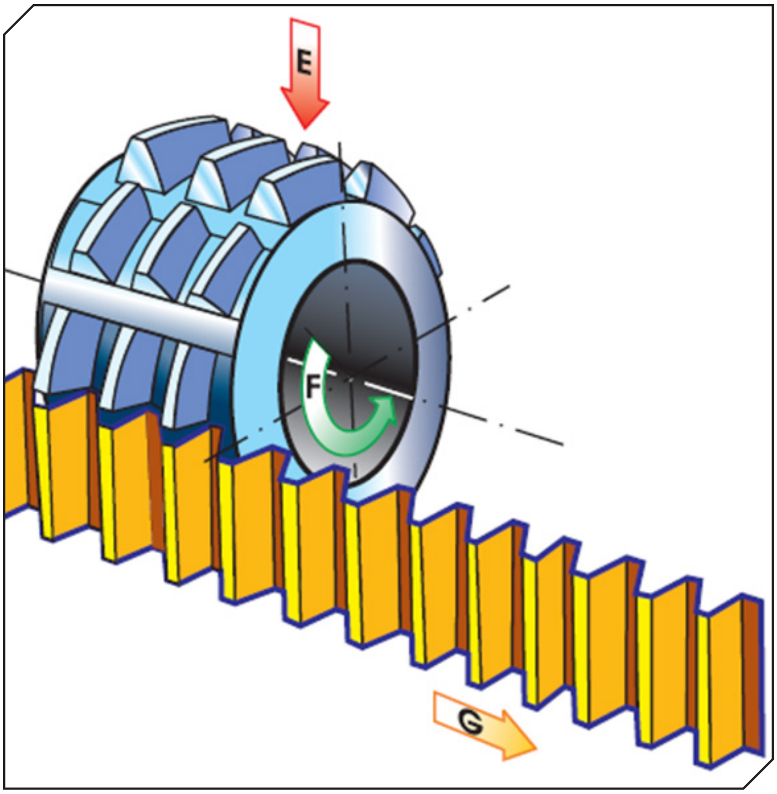

Figure 3 Hob cutter representing the trapezoidal generating profile.

20° for example, then the point where the tangent to the involute includes an angle of 20° to the line which connects this point with the center of the base circle is the pitch point. A larger pressure angle is realized merely by unrolling the cord further. The nominal pressure angle of a gear per definition is recorded at the pitch circle. However, the effective pressure angle below the pitch circle decreases as the distance to the base circle becomes smaller. Finally, at the base circle, the effective pressure angle is zero degree. The critical area is therefore at the beginning of the involute at the base circle. If the pressure angle is chosen very small, for example 15°, then the risk occurs that the dedendum (the distance from the pitch point to the root) reaches the base circle or even extends below the base circle.

The Mechanisms of Undercutting

Provided the gear is manufactured with a hob cutter, which has straight cutting edges and a rounding at the tip corners, then the hob cutter in Figure 3 represents the teeth of a trapezoidal generating profile and only the section of the cutting edge which stays above or at the base circle can form an involute profile (Ref. 1).

The angle of the trapezoid side walls is equal to the pressure angle. This means, if the pitch diameter is given by the product of the module and the number of teeth, then the base circle is automatically defined by the rules of the involute function:

Pitch Diameter cos (Pressure Angle) = Base Circle Diameter

In case of a depth factor larger than 1.00 and a large root clearance value, it is possible that the tip of the cutting edge protrudes below the base circle and will not generate any involute profile. To the contrary, the finish profile below the base circle is formed by the tip edge of the blade profile. The mathematical function it creates is a trochoid.

How the Trochoid Removes Part of the Involute

While the involute generation creates the flank surface points from the top of the tooth to the root, the trochoid generation works the opposite way. Figure 4 shows this generation in four steps. The green generating rack profile (equal to one cutting edge of the hob cutter) forms the upper involute section in position 1. A line which begins at the generated flank point and is perpendicular to the green line generating line, ends tangentially at the base circle. As the generating process progresses, the cutting-edge profile takes the position 2. The length of the green line towards the left side of the drawing is given by the center distance between the hob cutter and the gear blank. Already in position 2 it can be observed that the left end of the cutting edge penetrates below the base circle. From this point on, the blade tip begins to form a trochoidal root function, while in the same position the cutting edge towards the right side still forms the involute.

Cutting positions 3 and 4 show how the blade tip cuts deeper below the base

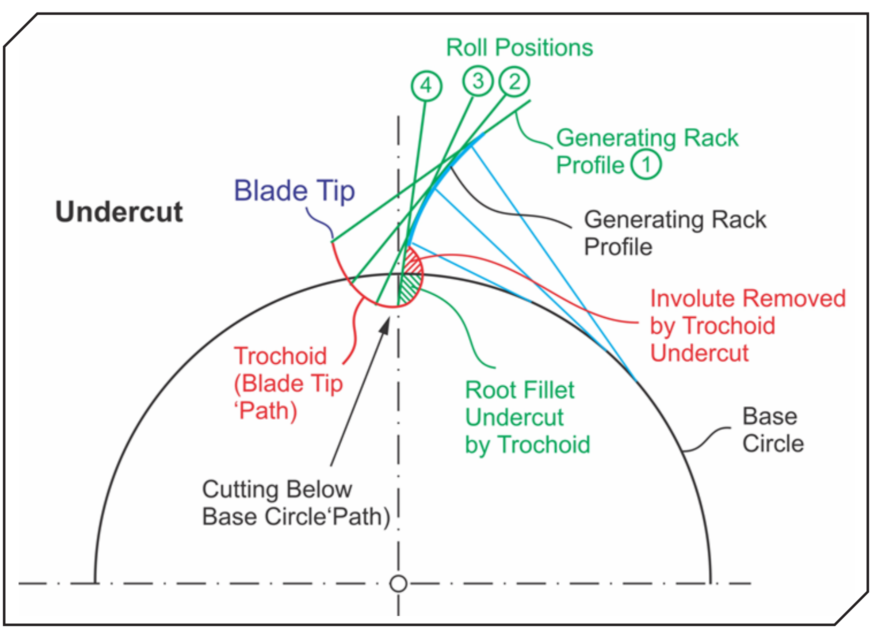

Figure 4 Generation of involute and trochoid.

circle and begins after position 4 to create undercut. Due to the length of the dedendum section of the blade, the undercut does not end below the base circle, but removes a considerable part of the already generated involute towards the root of the tooth.

Why is the Created Undercut Required for a Correct Meshing?

During the meshing process with the mating member, the top corner as shown in Figure 5 requires the room the undercut provides in order to roll without any interference through the entire mesh. Although the opposite flank of the red tooth in Figure 5 (left flank) is already in contact with the flank of the preceding tooth of the undercut pinion, the room the undercut provides in the circled area is required to avoid a collision.

The reason why the kinematic interaction between the two members is always as shown in Figure 5 is the fact that both members are generated with the same virtual generating rack. The generating rack is “only” virtual, but the hob cutter in Figure 3 represents the generating rack from the back in order to generate the pinion teeth, and another hob cutter (not shown in Figure 3) is positioned at the front of the generating rack to generate the mating gear. In other words, the virtual generating rack becomes a physical reality which provides the kinematic coupling which is necessary for an undisturbed meshing and the correct motion transmission between pinion and gear.

The graphic in Figure 6 shows a two-dimensional top-view onto the green generating rack with the blue pinion tooth being generated with undercut, and the red gear tooth (without undercut). The graphic shows that the edge corner of the red tooth comes very close to the generating rack corner which creates the undercut on the blue tooth. If two mating gears are generated according to the graphic in Figure 6, then there will be a small amount of clearance between the undercut root and the top edge corner of the blue mate (Ref. 2).

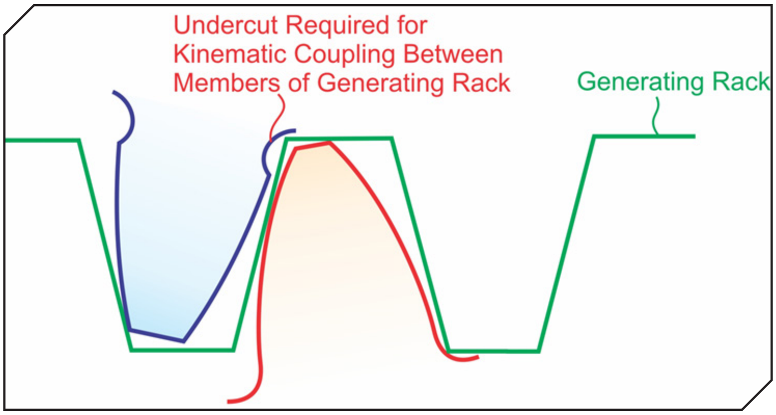

Figure 6 Kinematic link of two meshing teeth with the generating rack.

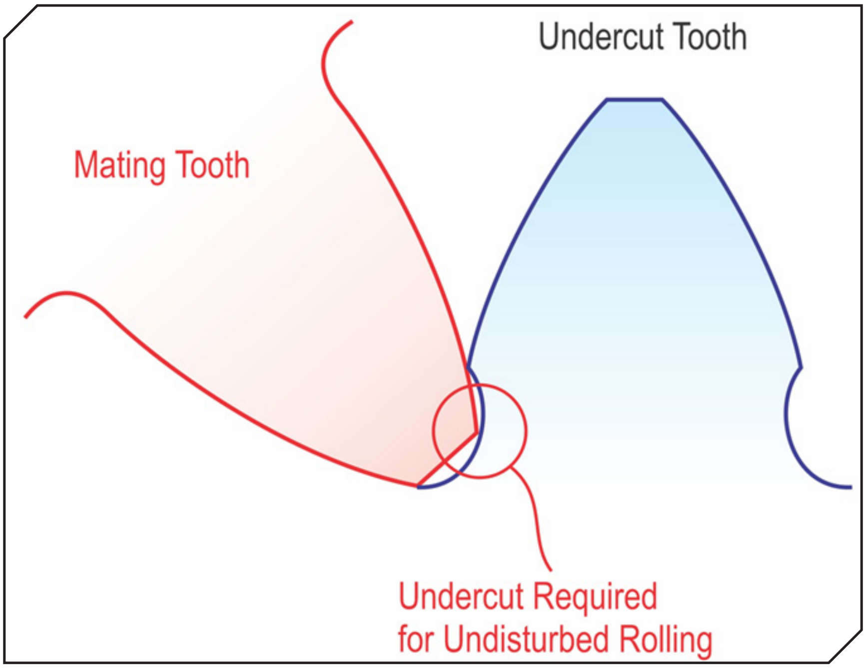

Figure 5 Interaction of undercut tooth with the mating tooth.

Possibilities to Eliminate Undercut

Gear engineers like to find ways to avoid an undercut condition. If the attempt is made to eliminate the undercut for example with a larger edge radius of the hob cutter blades, then an interference is created as shown in the top section of Figure 7. If the top edges of the mating tooth receive an adequate topland chamfer (bottom section), then the interference is eliminated. This is a procedure which is applied in cases where the undercut is not acceptable; for example, because of the reduced root bending strength.

The more common way to avoid undercut is applied at the beginning of a gearset design. Pinions with a low number of teeth show undercut due to the high angular rotation amount while being generated in a cutting machine. Therefore, most pinions below 17 teeth require a positive profile shift. The profile shift increases the center distance between the work gear and the manufacturing tool. Subsequently, the center distance between pinion and gear is also changed. If a center distance change is not desirable, then a so-called V0 profile shift is recommended. The V0 shift uses the same amount of positive profile shift applied to the pinion, with a negative sign for the gear. If the gear has a much higher number of teeth, then it will not be prone to undercut and will maintain a healthy profile and root fillet. The center distance in case of V0 gearing is identical to that without profile shift.

The effect of a positive profile shift is shown in Figure 8. The gear to the right is in mesh with its generating rack in a non-profile shifted configuration. It can be seen in the graphic that the root fillets show a beginning undercut. The positive profile shift at the left side gear increases the effective pressure angle of the involute because a part of the involute with a larger radius is used for the flank. Positive profile shifted gears do not have a smaller base circle, however, the root moves away from the base circle to a larger radius.

The profile shift factors of pinions and gears are often defined to optimize the sliding conditions within the flank surface, or they are used to balance the tooth thickness in the root between pinion and gear. This is a definite restriction if undercut is an issue. Small amounts of undercut do not present a problem in most cases. Small amounts of undercut can be seen as a way to realize a certain gear design and avoid interferences or rolling disturbances.

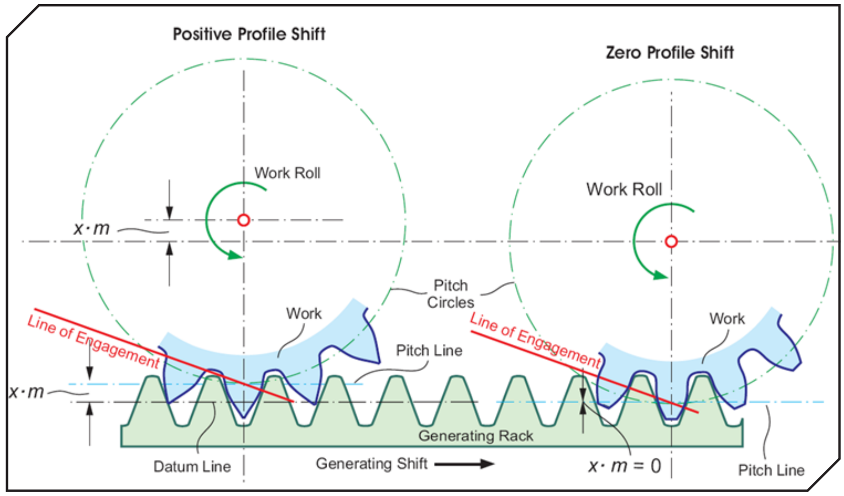

Figure 8 Elimination of undercut with profile shift.

Summary

Gear engineers like to avoid undercut, which is possible to some extent in the early design stages by choosing the right amount of profile shift. However, it is not always possible to eliminate undercut completely. In particular, if the number of pinion teeth is below 13, the profile shift factor is often not sufficient. If undercut is detected, this shows that the additional room in the root fillet transition to the flank is required for an undisturbed meshing process. This undercut means that the base circle is above the root diameter of the pinion. The top region of the meshing gear has a perfect involute which, however, cannot find an involute surface in the pinion root for a correct meshing. The undercut solves this problem partially by avoiding a metal-to-metal interference. Such an interference induces vibration and noise and creates small scratches and surface damage which can lead to crack propagation with the result of a tooth breakage.

If this undercut is eliminated merely by increasing the tool edge radius, an interference will occur. There is a possibility to eliminate the undercut by increasing the tool edge radius in combination with a correct dimensioned topland chamfer of the mating member. Because there is no software to aid in determining the required value of the tool edge radius increase and for the required topland chamfer dimension, this process requires experience as well as some trial-and-error loops.

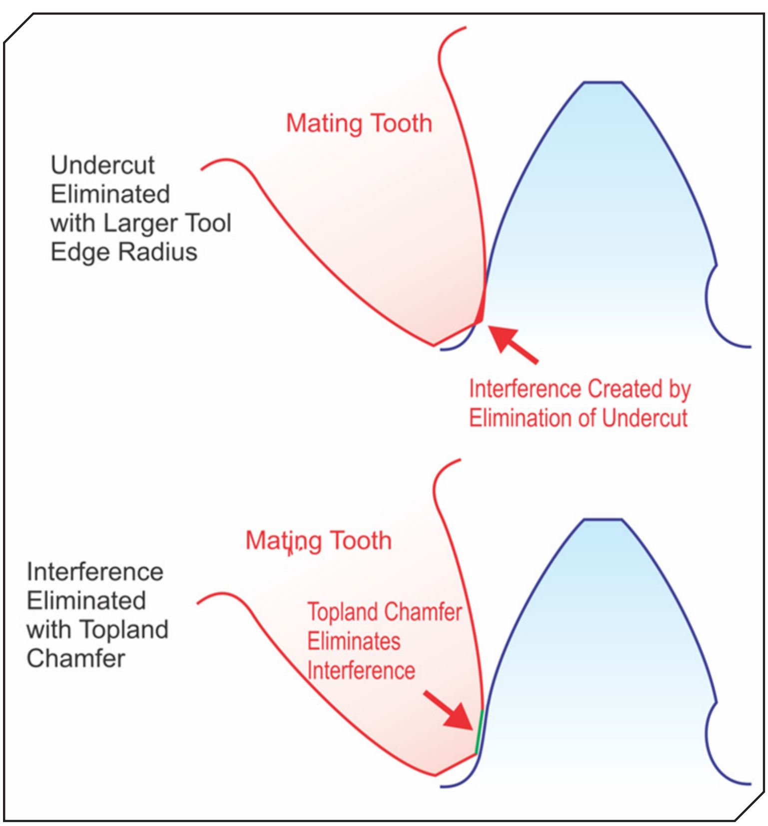

Figure 7 Elimination of Undercut (top) and elimination of interference (bottom).

References

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2. Stadtfeld, H.J., "*Practical Gear Engineering, Answers to Common Gear Manufacturing Questions*," Company Publication, The Gleason Works Rochester, New York, March 2019, Pages 107 to 108.

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